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RETAILER'S OPTIMAL PRICING AND ORDERING POLICIES FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH ORDER QUANTITY DEPENDENT TRADE CREDITS AND PARTIAL BACKLOGGING

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Abstract

In a competitive market, suppliers may offer different trade credit periods with different predetermined quantities to boost in their sales and to encourage retailers to order more quantities. In this article, we consider an inventory system with non-instantaneous deteriorating items, where the supplier provides the retailer with various trade credits linked to order quantity and the demand rate is considered to be deterministic depending on the selling price of the product. First, we develop a partial backlogged inventory model to identify the optimal pricing and ordering policies for retailer under various situations of trade credits. Numerical examples are presented to illustrate the proposed model. Finally, a sensitivity analysis is conducted to study the effects of main parameter values on the optimal solution and to draw managerial insights.

Keywords:

Inventory model;
Non-instantaneous deteriorating items;
Order quantity dependent trade credit;
Partial backlogging.

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1. Introduction

Many inventory practitioner assume that the deterioration of the items in an inventory system begins from the instant of their arrival in stock. However, there is a period of time during which most of the commodities such as firsthand vegetables and fruits, food grains, and medicines preserve their freshness and originality, that is, during which no deterioration occurs. After that period, these items will begin to deteriorate. Wu et al. [22] defined this phenomenon as ‘‘non-instantaneous deterioration’’. They also developed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Ouyang et al. [13] studied an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Yang et al. [23] developed the retailer’s optimal pricing and ordering policies for non-instantaneous deteriorating items with price-dependent demand. In this model, shortages are permitted and partially backlogged with a variable backlogging rate which depends on the waiting time for the next replenishment. Chang et al. [3] proposed optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. Geetha and Uthayakumar [7] developed an economic design of an inventory policy for non-instantaneous deteriorating items with permissible delay in payments under partial backlogging. Maihami and Kamalabadi [10] presented a joint pricing and inventory control model for non-instantaneous deteriorating items with both price and time dependent demand under partial backlogging. Soni [15] derived optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment. By considering all possible replenishment cycle time , Wu et al. [21] derived optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment. Wang et al. [20] presented a dynamic pricing policy for non-instantaneous deteriorating items. Tsao [17] modeled a joint location, inventory and preservation decision-making policy for non-instantaneous deteriorating items under credit period provided by an outside supplier to the wholesaler which has a distribution system with distribution centers. Tsao [18] also considered the problem of ordering non-instantaneously deteriorating products under price adjustment and trade credit. Mashud et al. [11] derived an inventory model for non-instantaneous deteriorating item having different deterioration rates with stock and price dependent demand by allowing partial backlogging. Soni et al. [16] developed a vendor managed inventory model for non-instantaneous deteriorating item having generalized time dependent deterioration rate with quadratic demand with respect to time and permitting shortages with partial backlogging.

In today’s business transactions, the supplier may offer a permissible delay in payments to encourage the retailer to order large quantities, but require immediate payment for small quantities. Hence, the supplier may decide a predetermined order quantity above which delay in payments is permitted and below which delay in payment is not permitted. This type of delay in payments is known as order-quantity-dependent trade credit. Khouja and Mehrez [8] investigated the effect of supplier credit policies on the optimal order quantity. They discussed two types of supplier credit policies: the first type is one in which trade credits are independent of the order quantity, and the second type is one in which trade credits are linked to the order quantity. Shinn and Hwang [14] developed optimal pricing and ordering policies for retailers simultaneously in the case of an order-size-dependent delay in payments. Chang et al. [2] developed an EOQ model for deteriorating items, where suppliers link credit to order quantity. Chung and Liao [5] discussed the optimal replenishment cycle time for an exponentially deteriorating item under the delay in payments which depends on the quantity ordered. Chung et al. [6] derived the optimal inventory policies under permissible delay in payments depending on the ordering quantity. Liao [9] prepared a note on an EOQ model for deteriorating items under supplier credit linked to ordering quantity. Ouyang et al. [12] developed an EOQ model for deteriorating items with partially permissible delay in payments linked to order quantity. Chen et al. [4] derived a retailer's economic order quantity model when the supplier offers conditionally permissible delay in payments which link to order quantity. Chang et al. [1] developed an appropriate inventory model for non-instantaneously deteriorating items in circumstances where the supplier provides the retailer various trade credits linked to order quantity. Vandana and Sharma [19] developed an inventory model for retailers partial permissible delay in payment linked to order quantity with shortage which is partially backlogged.

In order to match more realistic situations, a non-instantaneous deteriorating inventory model for determining the retailer's optimal pricing and ordering policies with selling price dependent demand under various trade credits linked to order quantity is considered in our study. In this model, shortages are allowed and partially backlogged where the backlogging rate is constant and holding cost is assumed to be a linearly increasing function of time. The rest of the article is organized as follows. The assumptions and notations used in this model are presented in sections 2 and 3 respectively. In Section 4, a mathematical model is formulated to find the retailer's total profit function in various trade credit situations. We then develop the solution procedure for an optimal solution in section 5. In Section 6, numerical examples and graphical analysis are presented to demonstrate the developed model and the solution procedure. In section 7, sensitivity of the optimal solution with respect to system parameters is carried out and managerial insights are furnished for the retailer. Finally, we draw our concluding remarks in Section 8.

2. Assumptions

The following assumptions are used to develop the proposed inventory model.

1. There are a single retailer and a single supplier in the inventory system.
2. The customer demand is assumed to be deterministic depending on the selling price of the product. For simplicity, the customer demand rate $D(s)$ may be given by $D(s) = a - bs$, where a and b are non-negative constants.
3. Replenishment occurs instantaneously at an infinite rate and the lead time is zero.
4. There are no repair and replenishment of deteriorated items during the planning horizon.
5. Shortages are allowed and the backlogging rate is η which is a constant with $0 \le \eta < 1$ during stock out period.
6. To encourage the retailer to order more quantities, the supplier offers a permissible delay period M which links to the order quantity Q as follows:

i	M	Q
1	M_1	$0 \le Q < q_1$
2	M_2	$q_1 \le Q < q_2$
3	M_3	$q_2 \le Q < q_3$
⋮	⋮	⋮
n	M_n	$q_{(n-1)} \le Q < \infty$
Where $0 \le M_1 < M_2 < \cdots\cdots < M_n < \infty$		

7. The product has no deterioration during the time interval $[v_i, v_i + t_d]$ in each replenishment cycle when the permissible delay period is M_i, after which, the on-hand stocks deteriorate with time and the rate of deterioration is an increasing function of time defined as $\theta(t) = \beta t$, where $v_i + t_d \le t \le T_i$ and $0 \le \beta << 1$.
8. Holding cost is assumed to be a linearly increasing function of time. For simplicity, it may be given by $h(t) = h_1 + h_2 t$, where h_1 and h_2 are non-negative constants.

3. Notations

The following notations are used to develop the proposed inventory model.

Notation	Description
a, b	: Demand parameters
h_1, h_2	: Holding cost parameters
β	: Deterioration parameter
η	: Backlogging parameter
p	: Purchasing cost per unit
s	: Selling price per unit with $s \ge p$
k	: Shortage cost per unit per unit time
l	: Lost sales cost per unit per unit time
d	: Deterioration cost per unit per unit time
O	: Ordering cost per order
$I_{i1}(t)$	: Inventory level at any time t during the time interval $[0, v_i]$
$I_{i2}(t)$	: Inventory level at any time t during the time interval $[v_i, v_i + t_d]$
$I_{i3}(t)$	: Inventory level at any time t during the time interval $[v_i + t_d, T_i]$
IP	: Maximum positive stock level at $t = v_i$
IB	: Maximum backordered quantity during stock out period
Q_i	: Order quantity per replenishment cycle when the permissible delay period is M_i
v_i	: The time at which the retailer receives the order when the permissible delay period is M_i
T_i	: The length of replenishment cycle when the permissible delay period is M_i
I_e	: Rate of interest earned
I_c	: Rate of interest charged

M_i	: The permissible delay period offered by the supplier when the retailer orders Q_i quantity where $q_{(i-1)} \le Q < q_i$

4. Mathematical Formulation of the Proposed Inventory Model

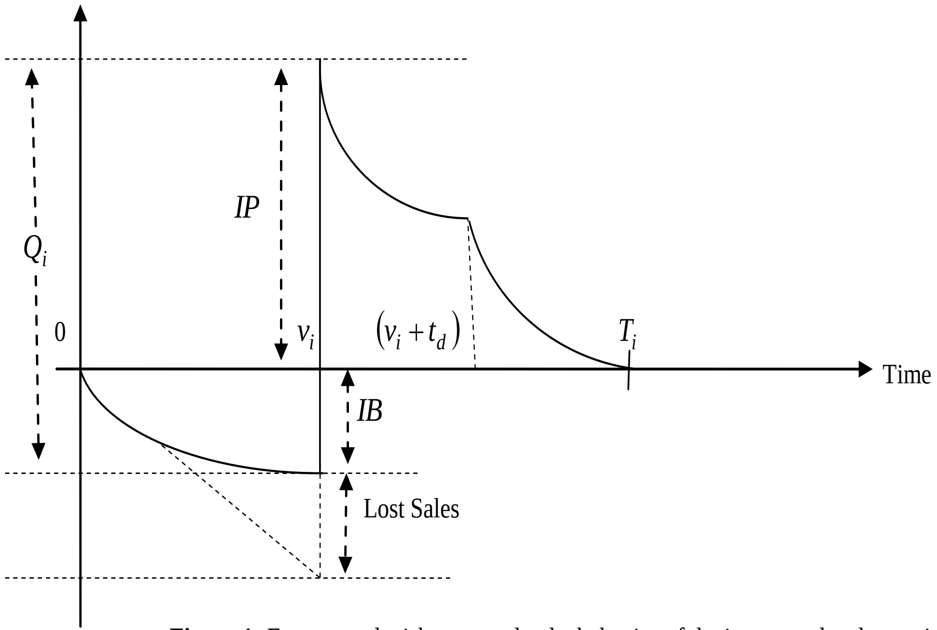


Figure 1: For one replenishment cycle, the behavior of the inventory level over time

For a given delay in payment time M_i, to determine the inventory level at any time $t \in [0, T_i]$, suppose that the inventory level is initially (i.e. at time $t = 0$) zero. At time $t = 0$, the shortage starts and some sales will be lost due to partial backlogging and other will accumulate up to the level IB during the time $[0, v_i]$. The retailer will place the order of quantity Q_i units at time $t = v_i$ and receive instantaneously at an infinite rate from supplier and retailer will firstly clear the previous backlog. At time $t = v_i$, the remaining positive stock will be IP which will be depleted as follow: The product has no deterioration and the inventory level will gradually decrease due to demand only during the time interval $[v_i, v_i + t_d]$ and during the time interval $[v_i + t_d, T_i]$, the inventory level will also decrease due to both demand and deterioration until it will reduce to zero. Each cycle will be completed with zero stock at time $t = T_i$ and the enire process will repeat itself. The retailer's inventory for one replenishment cycle is shown in Figure 1. Therefore, differential equations governing the proposed model are as follows:

$$I'_{i1}(t) = -\eta D(s), \qquad 0 \le t \le v_i \tag{1}$$

$$I'_{i2}(t) = -D(s), \qquad v_i \le t \le v_i + t_d \tag{2}$$

$$I'_{i3}(t) = -\theta(t) I_{i3}(t) - D(s), \qquad v_i + t_d \le t \le T_i \tag{3}$$

With the boundary conditions $I_{i1}(0)=0, I_{i2}(v_i)=IP$, and $I_{i3}(T_i)=0$.

The solutions of these linear differential equations are given by

$$I_{i1}(t)=-\eta(a-bs)t, \qquad 0\le t\le v_i \tag{4}$$

$$I_{i2}(t)=IP-(a-bs)(t-v_i), \qquad v_i\le t\le v_i+t_d \tag{5}$$

$$I_{i3}(t)=(a-bs)\left\{(T_i-t)+\frac{\beta}{6}\left(T_i^3-3T_it^2+2t^3\right)\right\}, \qquad v_i+t_d\le t\le T_i \tag{6}$$

The maximum backordered quantity during the stock out period is given by

$$IB=-I_{i1}(v_i)=\eta(a-bs)v_i \tag{7}$$

By the continuity of the inventory level functions $I_{i2}(t)$ and $I_{i3}(t)$ at the point $t=v_i+t_d$, one can get the maximum positive stocks at $t=v_i$

$$IP=(a-bs)\left\{(T_i-v_i)+\frac{\beta}{6}\left(T_i^3-3T_i(v_i+t_d)^2+2(v_i+t_d)^3\right)\right\} \tag{8}$$

So, the order quantity for each cycle is

$$Q_i=IB+IP=(a-bs)\left\{(T_i-(1-\eta)v_i)+\frac{\beta}{6}\left(T_i^3-3T_i(v_i+t_d)^2+2(v_i+t_d)^3\right)\right\} \tag{9}$$

From equations (5) and (8), one can get

$$I_{i2}(t)=(a-bs)\left\{(T_i-t)+\frac{\beta}{6}\left(T_i^3-3T_i(v_i+t_d)^2+2(v_i+t_d)^3\right)\right\}, \qquad v_i\le t\le v_i+t_d \tag{10}$$

The total profit of the retailer per unit time is composed of sales revenue, ordering cost, purchasing cost, deterioration cost, holding cost, shortage cost, lost sales cost, interest earned and interest charged. These components are evaluated as follows:

a) Sales Revenue: The retailer's sales revenue per unit time is given by

$$SR=\frac{s}{T_i}\left\{IB+\int_{v_i}^{T_i}D(s)dt\right\}=\frac{s(a-bs)}{T_i}\{T_i-(1-\eta)v_i\} \tag{11}$$

b) Ordering Cost: The ordering cost per unit time is $OC=\dfrac{O}{T_i}$ (12)

c) Purchasing Cost: The purchasing cost per unit time is given by

$$PC=\frac{pQ_i}{T_i}=\frac{p(a-bs)}{T_i}\left\{(T_i-(1-\eta)v_i)+\frac{\beta}{6}\left(T_i^3-3T_i(v_i+t_d)^2+2(v_i+t_d)^3\right)\right\} \tag{13}$$

d) Deterioration Cost: Since Q_i units are the order quantity per replenishment cycle when the permissible delay period is M_i, the deterioration cost per unit is given by

$$DC = \frac{d}{T_i}\left\{ I_{i2}(v_i + t_d) - \int_{v_i + t_d}^{T_i} D(s)dt \right\}$$

$$DC = \frac{d(a-bs)\beta}{6T_i}\left\{ T_i^3 - 3T_i(v_i + t_d)^2 + 2(v_i + t_d)^3 \right\}$$

(14)

e) Holding Cost: The cost associated with the holding of the stocks per unit time is given by

$$HC = \frac{1}{T_i}\left\{ \int_{v_i}^{v_i + t_d} (h_1 + h_2 t) I_{i2}(t)dt + \int_{v_i + t_d}^{T_i} (h_1 + h_2 t) I_{i3}(t)dt \right\}$$

$$HC = \frac{(a-bs)}{T_i}\left\{ \begin{array}{l} h_1\left\{ \frac{1}{2}(T_i - v_i)^2 + \frac{\beta}{12}\left(\begin{array}{l} T_i^4 - 2T_i^3 v_i + 2T_i\left(v_i^3 - 3v_i t_d^2 - 2t_d^3\right) \\ + \left(3t_d^4 + 6v_i^2 t_d^2 + 8v_i t_d^3 - v_i^4\right) \end{array} \right) \right\} \\ + h_2\left\{ \frac{1}{6}\left(T_i^3 - 3T_i v_i^2 - 2v_i^3\right) + \frac{\beta}{120}\left\{ \begin{array}{l} 3T_i^5 - 10T_i^3 v_i^2 + 15T_i(v_i + t_d)^2 \\ \left(t_d^2 + 2v_i t_d - v_i^2\right) \\ + 4(v_i + t_d)^3\left(3t_d^2 + 6v_i t_d - 2v_i^2\right) \end{array} \right\} \right\} \end{array} \right\}$$

(15)

f) Shortage Cost: The shortage cost per unit time is calculated as

$$SC = \frac{k}{T_i}\int_0^{v_i} \{-I_{i1}(t)\}dt = \frac{k\eta(a-bs)}{2T_i}v_i^2$$

(16)

g) Lost Sales Cost: The shortages incurred in the initial phase of the cycle are partially lost. Consequently, the retailer has to bear an extra expense of the lost sale cost. Thus, the lost sale cost per unit time is given by

$$LSC = \frac{l}{T_i}\int_0^{v_i} (1-\eta)D(s)dt = \frac{l(a-bs)}{T_i}(1-\eta)v_i$$

(17)

Based on the values of the permissible delay M_i and the replenishment cycle T_i, there are two possible situations as follows:

Case 1: When the permissible delay time M_i is longer than or equal to the replenishment cycle T_i i.e. $T_i \le M_i$.

Case 2: When the permissible delay time M_i is shorter than the replenishment cycle T_i i.e. $M_i < T_i$

4.1 Case 1: When $T_i \le M_i$

In this case, the retailer will settle the account at time $t = M_i$. The retailer sold the backlogged stock to his customers at time $t = v_i$ and deposits the sales revenue in an interest bearing account. The retailer also sells the remaining stock during the time interval

$[v_i, T_i]$ and deposits the sales revenue in the same interest bearing account. Therefore, the total interest earned per unit time is given by

$$IE_1 = \frac{I_e}{T_i}\left\{ s(M_i - v_i)IB + s\int_{v_i}^{T_i} D(s)\,tdt + s(M_i - T_i)\int_{v_i}^{T_i} D(s)dt \right\}$$

$$IE_1 = \frac{I_e}{T_i} s(a - bs)\left\{ \eta(M_i - v_i)v_i + \frac{1}{2}\left(T_i^2 - v_i^2\right) + (M_i - T_i)(T_i - v_i) \right\}$$

(18)

Since the permissible delay time M_i is longer than or equal to the replenishment cycle T_i, there is no interest charged to the retailer. Hence, the total interest charged per unit time is given by

$$IC_1 = 0$$

(19)

Hence, the total profit of the retailer per unit time is given by

$$Z_1 = \{SR - OC - PC - DC - HC - SC - LSC - IC_1 + IE_1\}$$

(20)

4.2 Case 2: When $M_i < T_i$

Since the permissible delay time M_i is shorter than the replenishment cycle T_i, therefore, the total interest earned during the time interval $[v_i, M_i]$ is given by

$$IE_2[v_i, M_i] = I_e\left\{ s(M_i - v_i)IB + s\int_{v_i}^{M_i} D(s)\,tdt \right\}$$

$$IE_2[v_i, M_i] = sI_e(a - bs)\left\{ \eta(M_i - v_i)v_i + \frac{1}{2}\left(M_i^2 - v_i^2\right) \right\}$$

(21)

and the total sales revenue generated during the time interval $[v_i, M_i]$ is given by

$$SR_2[v_i, M_i] = s\left\{ IB + \int_{v_i}^{M_i} D(s)dt \right\} = s(a - bs)\{\eta v_i + (M_i - v_i)\}$$

(22)

Based on the difference in the total amount generated by sales revenue and earned interest on sales revenue during the time interval $[v_i, M_i]$ and the total purchasing cost of the items at the time $t = M_i$, two different subcases may arise:-

Subcase 2.1: When $SR_2[v_i, M_i] + IE_2[v_i, M_i] \geq pQ_i$

Subcase 2.2: When $SR_2[v_i, M_i] + IE_2[v_i, M_i] < pQ_i$

4.2.1 Subcase 2.1: When $SR_2[v_i, M_i] + IE_2[v_i, M_i] \geq pQ_i$

Since the total amount generated by sales revenue and earned interest on sales revenue during the time interval $[v_i, M_i]$ is greater than or equal to the total purchasing cost of the items at the time $t = M_i$, Therefore, the total interest earned per unit time will be

$$IE_{2.1}=\frac{1}{T_i}\left\{IE_2[v_i,M_i]+I_e(T_i-M_i)\{SR_2[v_i,M_i]+IE_2[v_i,M_i]-pQ_i\}+I_e s\int_{M_i}^{T_i}D(s)\,tdt\right\}$$

$$=\frac{(a-bs)I_e}{T_i}\left\{s\left\{\begin{matrix}\eta(M_i-v_i)v_i\\+\frac{1}{2}(T_i^2-v_i^2)\end{matrix}\right\}+(T_i-M_i)\left\{\begin{matrix}s\left\{\eta v_i+(M_i-v_i)+I_e\left\{\begin{matrix}\eta(M_i-v_i)v_i\\+\frac{1}{2}(M_i^2-v_i^2)\end{matrix}\right\}\right\}\\-p\left\{(T_i-(1-\eta)v_i)+\frac{\beta}{6}\left(\begin{matrix}T_i^3-3T_i(v_i+t_d)^2\\+2(v_i+t_d)^3\end{matrix}\right)\right\}\end{matrix}\right\}\right\}$$

(23)

and the total interest charged per unit time will be zero i.e.

$IC_{2.1}=0$

(24)

Hence, the total profit of the retailer per unit time is given by

$Z_{2.1}=\{SR-OC-PC-DC-HC-SC-LSC-IC_{2.1}+IE_{2.1}\}$

(25)

4.2.2 Subcase 2.2: When $SR_2[v_i,M_i]+IE_2[v_i,M_i]<pQ_i$

Since the total amount generated by sales revenue and earned interest on sales revenue during the time interval $[v_i,M_i]$ is less than the total purchasing cost of the items at the time $t=M_i$, Therefore, the total interest earned per unit time will be

$$IE_{2.2}=\frac{1}{T_i}\left\{IE_2[v_i,M_i]+I_e s\int_{M_i}^{T_i}D(s)\,tdt\right\}=\frac{s(a-bs)I_e}{T_i}\left\{\eta(M_i-v_i)v_i+\frac{1}{2}(T_i^2-v_i^2)\right\}$$

(26)

and the total interest charged on the unpaid payment per unit time will be

$$IC_{2.2}=\frac{I_c}{T_i}\left\{(T_i-M_i)(a-bs)\left\{\begin{matrix}p\left\{(T_i-(1-\eta)v_i)+\frac{\beta}{6}\left(T_i^3-3T_i(v_i+t_d)^2+2(v_i+t_d)^3\right)\right\}\\-s\left\{\eta v_i+(M_i-v_i)+I_e\left\{\eta(M_i-v_i)v_i+\frac{1}{2}(M_i^2-v_i^2)\right\}\right\}\end{matrix}\right\}\right\}$$

(27)

Hence, the total profit of the retailer per unit time is given by

$Z_{2.2}=\{SR-OC-PC-DC-HC-SC-LSC-IC_{2.2}+IE_{2.2}\}$

(28)

5. Solution Procedure

Our objective is to find the optimal selling price s, ordering time v_i, and ordering quantity Q_i that maximizes the total profit of the retailer $Z(v_i,s)$.

In order to maximize the total profit of the retailer per unit time with respect to the selling price s and

ordering time v_i, the necessary conditions are $\frac{\partial}{\partial v_i}Z(v_i,s)=0$ and $\frac{\partial}{\partial s}Z(v_i,s)=0$

(29)

The non-linearity of the equation (29) will not permit us to obtain the closed form solution. So, we recommend following solution procedure:

Step 1: For a given delay in payment time M_i, assign numerical values to all inventory parameters except s and v_i.

Step 2: Solve simultaneous equations in (29) with the help of the mathematical software. Here, we have used Mathematica 5.2.

Step 3: First test the feasibility conditions.

Step 4: If feasibility conditions are satisfied, then test sufficient conditions which are

$$\frac{\partial^2 Z}{\partial v_i^2}<0,\ \frac{\partial^2 Z}{\partial s^2}<0,\text{ and }\left(\frac{\partial^2 Z}{\partial v_i^2}\times\frac{\partial^2 Z}{\partial s^2}\right)-\left(\frac{\partial^2 Z}{\partial v_i \partial s}\right)^2>0.$$

Step 5: If sufficient conditions are satisfied, then find out the total profit of the retailer per unit time for each case by using equations (20), (25), (28) respectively and order quantity Q_i for each case using equation (9).

6. Numerical Examples and Graphical Analysis

In order to illustrate the above model and its solution procedure, we consider the following numerical examples which cover all cases that arise in the model.

6.1 Example 1

In order to illustrate case 1 of the proposed model, we consider an inventory system with the following numerical data in the proper unit:

$a=600, b=0.8, h_1=10, h_2=0.5, \beta=0.01, \eta=0.9, p=300, k=20, l=25, d=15, O=500,$

$T_i=10, t_d=1, I_e=0.06, I_c=0.09, \text{and } M_i=12.$

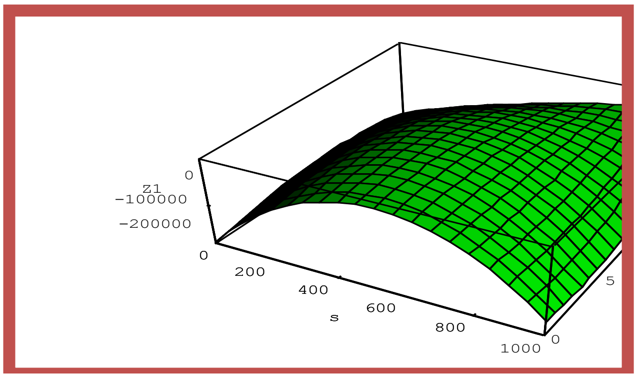
Table 1 Permissible delay period M_i offered by the supplier for example 1

M_i	Q_i
6	$0\le Q_1<1400$
12	$1400\le Q_i<2800$
18	$2800\le Q_i<4200$
24	$4200\le Q_i<\infty$

we obtain the following optimal results in the proper units:

$$s=496.00,\ v_i=4.1656,\ Q_i=2108.28,\text{ and }Z_1=74918.20.$$

Figure 2 Concavity of the Retailer's total profit function Z_1 with respect to v_i and s



6.2 Example 2

In order to illustrate subcase 2.1 of the model, we consider an inventory system with the same numerical data which used in example 1 except the value of M_i.

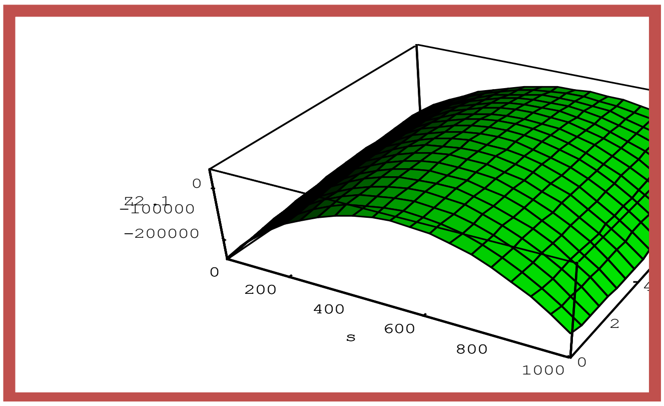
Table 2 Permissible delay period M_i offered by the supplier for example 2

M_i	Q_i
5	$0 \le Q_1 < 1400$
8	$1400 \le Q_i < 2800$
11	$2800 \le Q_i < 4200$
14	$4200 \le Q_i < \infty$

By taking $M_i = 8$, we obtain the following optimal results in the proper units:

$$s = 513.87,\ v_i = 3.6078,\ Q_i = 1996.79,\ and\ \ Z_{2.1} = 63766.50.$$

Figure 3 Concavity of the Retailer's total profit function $Z_{2.1}$ with respect to v_i and s



6.3 Example 3

In order to illustrate subcase 2.2 of the model, we consider an inventory system with the same numerical data which used in example 1 except the value of M_i.

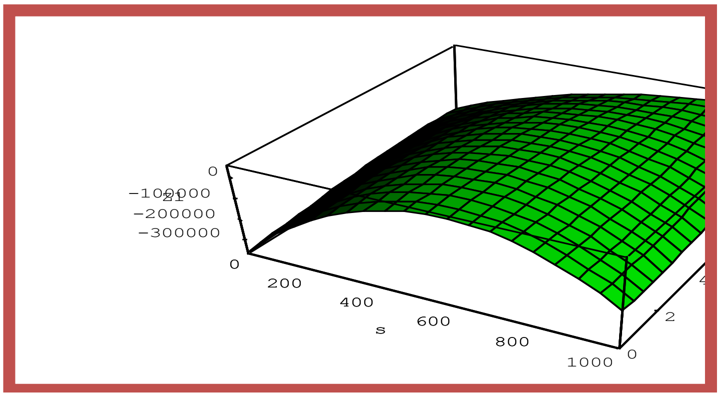
Table 3 Permissible delay period M_i offered by the supplier for example 3

M_i	Q_i
2	$0 \le Q_1 < 1000$
4	$1000 \le Q_i < 2000$
6	$2000 \le Q_i < \infty$

By taking $M_i = 4$, we obtain the following optimal results in the proper units:

$$s = 559.47,\ v_i = 2.1839,\ Q_i = 1684.13,\ and\ \ Z_{2.2} = 44069.50.$$

Figure 4 Concavity of the Retailer's total profit function $Z_{2.2}$ with respect to v_i and s



7. Sensitivity Analysis and Managerial Insights

In this section, a sensitivity analysis is performed to analyze the effects of various system parameters on the selling price of the product, order quantity, and the retailer's total profit per unit time of the system. The previous numerical example 1 is used for this sensitivity analysis. The sensitivity analysis is performed by changing each value of the parameter by -20%, -10%, +10%, and +20% while keeping remaining parameters unchanged. The computational results of this analysis are shown in Table 4.

Table 4

Parameter	% Change in the value of Parameter	s	v_i	Q_i	Z_1
a	-20	420.51	4.3909	1478.53	37214.80
	-10	458.26	4.2712	1792.61	54425.20
	0	496.00	4.1656	2108.28	74918.20
	+10	533.73	4.0718	2425.28	98699.80
	+20	571.45	3.9880	2743.40	125775.00
b	-20	590.31	3.9494	2322.24	112429.00
	-10	537.92	4.0620	2214.53	91385.90
	0	496.00	4.1656	2108.28	74918.20
	+10	461.69	4.2611	2003.38	61772.70
	+20	433.09	4.3493	1899.67	51114.80
h_1	-20	494.95	4.0313	2126.58	75720.50
	-10	495.48	4.0994	2117.31	75314.20
	0	496.00	4.1656	2108.28	74918.20
	+10	496.50	4.2300	2099.56	74532.14
	+20	496.98	4.2926	2091.16	74155.60
h_2	-20	495.92	4.0996	2113.64	75051.70
	-10	495.96	4.1322	2110.99	74983.80
	0	496.00	4.1656	2108.28	74918.20
	+10	496.02	4.1997	2105.69	74855.00
	+20	496.04	4.2345	2103.05	74794.40
β	-20	494.45	3.9853	2099.82	76075.60
	-10	495.24	4.0757	2104.40	75487.40
	0	496.00	4.1656	2108.28	74918.20
	+10	496.71	4.2548	2111.66	74367.90
	+20	497.39	4.3433	2114.41	73836.00
η	-20	498.85	3.6456	1989.31	68869.70
	-10	497.20	3.9287	2043.58	71829.80
	0	496.00	4.1656	2108.28	74918.20
	+10	495.13	4.3676	2181.26	78106.90
	+20	494.52	4.5425	2260.82	81375.60
p	-20	474.90	4.0119	2295.25	88127.00
	-10	485.45	4.0903	2201.42	81382.40
	0	496.00	4.1656	2108.28	74918.20
	+10	506.53	4.2380	2015.95	68732.20

	+20	517.07	4.3077	1924.14	62822.30
k	-20	494.49	4.3201	2109.78	75578.40
	-10	495.26	4.2414	2109.02	75241.80
	0	496.00	4.1656	2108.28	74918.20
	+10	496.69	4.0924	2107.74	74606.90
	+20	497.36	4.02168	2107.17	74606.90
l	-20	495.91	4.1707	2108.66	74960.60
	-10	495.96	4.1681	2108.43	74939.40
	0	496.00	4.1656	2108.28	74918.20
	+10	496.04	4.1631	2108.12	74897.10
	+20	496.08	4.1605	2107.98	74875.90
Table 4 Continued					
d	-20	495.93	4.1585	2109.36	74966.50
	-10	495.96	4.1620	2108.87	74942.40
	0	496.00	4.1656	2108.28	74918.20
	+10	496.03	4.1691	2107.78	74894.10
	+20	496.06	4.1727	2107.28	74870.00
O	-20	496.00	4.1656	2108.28	74928.20
	-10	496.00	4.1656	2108.28	74913.20
	0	496.00	4.1656	2108.28	74918.20
	+10	496.00	4.1656	2108.28	74913.20
	+20	496.00	4.1656	2108.28	74908.20
T_i	-20	486.98	3.02453	1708.64	82970.40
	-10	491.35	3.5795	1910.59	78977.60
	0	496.00	4.1656	2108.28	74918.20
	+10	500.90	4.7852	1910.59	70812.40
	+20	506.90	4.7852	2486.11	66683.50
t_d	-20	496.54	4.1665	2113.87	74595.90
	-10	496.27	4.1664	2111.05	74757.00
	0	496.00	4.1656	2108.28	74918.20
	+10	495.73	4.1641	2105.55	75079.40
	+20	495.46	4.1618	2102.88	75240.30
I_e	-20	504.40	4.3364	2026.83	65066.40
	-10	500.07	4.2438	2069.03	69963.70
	0	496.00	4.1656	2108.28	74918.20
	+10	492.17	4.0986	2144.90	79923.10
	+20	488.57	4.0406	2179.10	74972.60

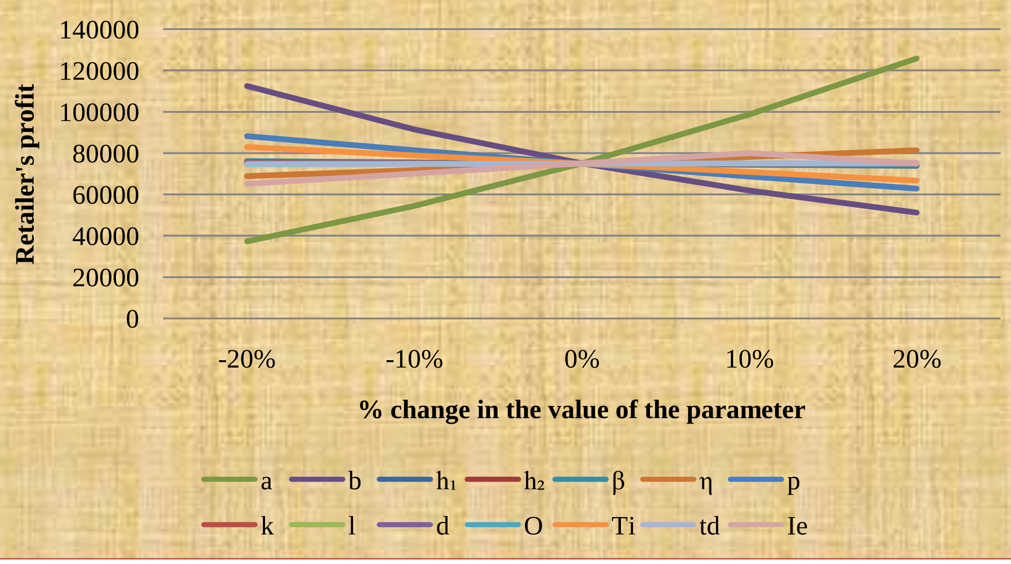
Based on the computational results shown in Table 4, the following managerial insights are obtained.

1. Increase in the demand parameter a results in an increase in the selling price s of the product, the order quantity Q_i, and the retailer's total profit per unit time of the system. Thus the demand parameter a has a major impact on the retailer's total profit.
2. Increase in the demand parameter b results in a decrease in the selling price s of the product, the order quantity Q_i, and the retailer's total profit per unit time of the system.

Hence, if the retailer can effectively decrease the demand parameter b, the retailer's total profit will be increased sufficiently.

3. Increasing the backlogging parameter η or equivalently decreasing the backlogging rate decreases the selling price s of the product, but increases the order quantity Q_i and the retailer's total profit per unit time of the system.
4. It can be seen that when the deterioration parameter β increases, the selling price s of the product and the order quantity Q_i increase while the retailer's total profit decreases. Hence, if the retailer can effectively reduce the deteriorating rate of item by improving equipment of his warehouses, the retailer's total profit will be increased sufficiently.
5. The order quantity Q_i and the retailer's total profit per unit time of the system will decrease with an increase in the values of the parameters $p, k, l, d,$ and t_d, but increase with an increase in the value of the parameter I_e. The selling price s of the product will increase with an increase in the values of the parameters $p, k, l,$ and d, but decrease with an increase in the value of the parameters I_e and t_d.
6. Increase in the holding cost parameters h_1 and h_2 results in an increase in the selling price s of the product, but a decrease in the order quantity Q_i and the retailer's profit. This implies that when the holding cost is higher the retailer's total profit is low.

Figure. 5: Change in the retailer's profit w. r. t. % change in the value of the parameter



8. Concluding Remarks

In this paper, we have developed an inventory model for non-instantaneous deteriorating items in circumstances where the supplier provides the retailer various trade credits linked to order quantity and the demand rate is considered to be deterministic depending on the selling price of the product. Apart from the above features, we have also considered time dependent deterioration rate and holding cost which generalizes the proposed model. A solution procedure has been developed for the proposed model to maximize the retailer's total profit per unit of the system. Furthermore, we have provided numerical examples and conducted a sensitivity analysis to illustrate the proposed model. There is enough scope to extend this model by incorporating inflation, price discounts, and other factors.

References

[1] Chang, C.T., Cheng, M.K. and Ouyang, L.Y., "Optimal pricing and ordering policies for non-instantaneously deteriorating items under order-size-dependent delay in payments", *Applied Mathematical Modelling*, Vol. 39, pp. 747-763, 2015.

[2] Chang, C.T., Ouyang, L.Y. and Teng, J.T., "An EOQ model for deteriorating items under supplier credits linked to ordering quantity", *Applied Mathematical Modelling*, Vol. 27, pp. 983-996, 2003.

[3] Chang, C.T., Teng, J.T. and Goyal, S.K., "Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand", *International Journal of Production Economics*, Vol. 123, pp. 62-68, 2010.

[4] Chen, S.C., Cárdenas-Barrón, L.E. and Teng, J.T., "Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity", *International Journal of Production Economics*, Vol. 155, pp. 284-291, 2014.

[5] Chung, K.J. and Liao, J.J., "Lot-sizing decisions under trade credit depending on the ordering quantity", *Computers & Operations Research*, Vol. 31, pp. 909-928, 2004.

[6] Chung, K.J., Goyal, S.K. and Huang, Y.K., "The optimal inventory policies under permissible delay in payments depending on the ordering quantity", *International Journal of Production Economics*, Vol. 95, pp. 203-213, 2005.

[7] Geetha, K.V., and Uthayakumar, R., "Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments", *Journal of Computational and Applied Mathematics*, Vol. 233, No. 10, pp. 2492-2505, 2010.

[8] Khouja, M. and Mehrez, A., "Optimal inventory policy under different supplier credit policies", *Journal of Manufacturing Systems*, Vol. 15, No. 5, pp. 334-339, (1996).

[9] Liao, J.J., "A note on an EOQ model for deteriorating items under supplier credit linked to ordering quantity", *Applied Mathematical Modelling*, Vol. 31, pp. 1690-1699, 2007.

[10] Maihami, R. and Kamalabadi, I.N., "Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand", *International Journal of Production Economics*, Vol. 136, pp. 116-122, 2012.

[11] Mashud, A., Khan, M., Uddin, M. and Islam, M., "A non-instantaneous inventory model having different deterioration rates with stock and price dependent demand under partially backlogged shortages', *Uncertain Supply Chain Management*, Vol. 6, No. 1, pp. 49-64, 2018.

[12] Ouyang, L.Y., Teng, J.T., Goyal, S.K. and Yang, C.T., "An economic order quantity model for deteriorating items with partially permissible delay in payments linked to order quantity", *European Journal of Operations Research*, Vol. 194, pp. 418-431, 2009.

[13] Ouyang, L.Y., Wu, K.S. and Yang, C.T., "A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments", *Computers and Industrial Engineering*, Vol. 51, pp. 637-651, 2006.

[14] Shinn, S.W. and Hwang, H., "Optimal pricing and ordering policies for retailers under order size-dependent delay in payments", *Computers & Operations Research*, Vol. 30, pp. 35-50, 2003.

[15] Soni, H.N., "Optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment", *International Journal of Production Economics*, Vol. 146, No. 1, pp. 259-268, 2013.

[16] Soni, H.N., Gor, A.J. and Patel, H.R., "Vendor managed inventory model for non-instantaneous deteriorating product with quadratic demand allowing partial backlogging", *Uncertain Supply Chain Management*,Vol. 6, No. 3, pp. 321-334, 2018.

[17] Tsao, Y.C., "Joint location, inventory, and preservation decisions for non-instantaneous deterioration items under delay in payments", *International Journal of Systems Science*, Vol. 47, No. 3, pp. 572-585, 2016.

[18] Tsao, Y.C., "Ordering policy for non-instantaneously deteriorating products under price adjustment and trade credits", *Journal of Industrial & Management Optimization*, Vol. 13, No. 1, pp. 329-347, 2017.

[19] Vandana and Sharma, B.K., "An EOQ model for retailers partial permissible delay in payment linked to order quantity with shortages", *Mathematics and Computers in Simulation*, Vol. 125, pp. 99-112, 2016.

[20] Wang, Y., Zhang, J. and Tang, W., "Dynamic pricing for non-instantaneous deteriorating items", *Journal of Intelligent Manufacturing*, Vol. 26, No. 4, pp. 629-640, 2015.

[21] Wu, J., Skouri, K., Teng, J.T. and Ouyang, L. Y., "A note on optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment", *International Journal of Production Economics*, Vol. 155, pp. 324-329, 2014.

[22] Wu, K.S., Ouyang, L.Y. and Yang, C.T., "An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging", *International Journal of Production Economics*, Vol. 101, pp. 369-384, 2006.

[23] Yang, C.T., Ouyang, L.Y. and Wu, H.H., "Retailer's optimal pricing and ordering policies for non-instantaneous deteriorating items with price-dependent demand and partial backlogging", *Mathematical Problems in Engineering*, Vol. 2009, Article ID 198305, 18 pages, 2009.